Step By Step Model Drawing
Discover the Ah Ha Moments

Dawn Sparks
Luke Matlack
Learning Targets

- I understand how mathematical modeling (tape diagrams) builds coherence, perseverance, and reasoning abilities in students.
- I understand how using tape diagrams shift students to be more independent learners.
- I can model problems that demonstrates the progression of mathematical modeling throughout Common Core State Standards.
What are the unique needs of...

- Migrant Students?
- ELL Students?
Seven Areas of Concern for Migrant Families

- Migrant Lifestyle Richness and Opportunity: Bicultural Dual Language Global Competency
- Access to Services
- Health
- Instructional Time
- Educational Support in the Home
- School Engagement

- Minority Status
- English Language Development
- Poverty
- Mobility
- Parents/Little Formal Schooling
- Social and Cultural Processes
- Bicultural Ambivalence
- Acculturation or Assimilation or Oppositional Identity

Stereotypes
Sample Problem: Try It

- The sum of two numbers is 36. The small number is one-third of the larger number. Find the two numbers.
Sharing Strategies…

- How did you solve the problem?

- Share your ideas with a partner…

- Ideas from the group-what were some of the strategies you used?
How did you solve your problem?

Concrete

Pictorial

Abstract

$9 + 6 = 15$
How many ways can you describe an orange...

- Make a list of all the words you can use to describe the orange in front of you...

- Use all of your senses in your description...
How many ways can you describe an orange...

- Now think of all the words you can use to describe the model of the orange

- What words would you not have used if you only used the model.

- Cross out any words that you would not have used in your first list.
What words can you now use to describe the word orange.

Continue to cross out all of the words you could no longer use to describe the word you just used.
Making the connection ...
Typical Instructional Sequence

Concrete
Model with Objects

Abstract
Representing problem situations with equations

What’s missing from this sequence?
Ali has 4 toy cars. David has 3 toy cars. How many toy cars do they have together?

Concrete
Use concrete objects to form two groups and put the two groups together.
1,2,3 4
1,2,3
1,2,3,4,5,6,7...7 cars

Abstract
4 + 3 = 7
Ali has 4 toy cars. David has 3 toy cars. How many toy cars do they have together?

**Concrete**

Use concrete objects to form two groups and put the two groups together.

1,2,3 4

1,2,3 4,5,6,7...7 cars

**Representational**

7 cars

4 5,6,7

**Abstract**

4 + 3 = 7

Abstracting to another level.

7 cars

4 5,6,7
Progression of Understanding

Developing Conceptual Understanding
Concrete → Pictorial → Visualization → Abstract

2 apples + 5 apples = 7 apples
2 + 5 = 7
S = J - 3
S + J = 7
**Representational Math Drawings**

Drawing pictures that represent concrete objects provides a bridge to help children connect their concrete representations to the abstract world of mathematical symbols.

“Math drawings facilitate reflection and discussion because they remain after the problem has been solved.”

(OA Progressions, p. 8)

Children need **many** opportunities to create such drawings.
Tape Diagrams
CCSSM Suggested Math Drawing: Tape Diagram

What is a tape drawing?

A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as strip diagrams, bar model, fraction strip, or length model.

(CCSSM Glossary, p. 87)
Where are tape diagrams in the Common Core Math Standards?

What is a Tape Diagram?

Grade 1: Math Drawings (1.OA.1, 1.OA.2)
Grade 2: Math Drawings (2.OA.1, 2.OA.2, 2.MD.5)
Grade 3: Visual Fraction Model (3.NF.3a-d)
Grade 4: Visual Fraction Model (4.NF.3, 4.NF.4, 4.OA.2)
Grade 5: Visual Fraction Model (5.NF.2-4, 6, 7)
Grade 6: Tape Diagrams (6.RP-3) Visual Fraction Model (6.NS-1)
Grade 7: Visual Model for Problem Solving (7RP1-3) Number Line Diagram (7.NS-1)
Forms of a Tape Diagram

- **Part-Whole Model**
  - Also known as the ‘part-part-whole’ model, shows the various parts which make up a whole

- **Comparison Model**
  - Shows the relationship between two quantities when they are compared
Forms of the Tape Diagram

Part Whole Model
Whole
Part

Fraction Model
3 pieces of size one-third

Additive Comparison Model
smaller quantity

difference

larger quantity

Models for Ratios & Multiplicative Comparison
4 times as many as; a 1:4 ratio
Making the Case…
Focus on number sense!

Research indicates that early number sense predicts school success more than other measures of cognition like verbal, spatial or memory skills or reading ability.

Jordoan, Kaplan, Locuniak, and Ramineni, 2007
What is a tape drawing?

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(CCSSM Glossary, p. 87)
Model Drawing Let’s Get Started-

- **Learn the process** or how to use model drawings to solve word problems
- **Apply the process** independently when solving word problems
STEPS for Model Drawing...

1. Teacher reads real world problem
2. Students listen and visualize the problem
3. Students write down the key facts
   a) who, what, how
4. Students write a mmm statement
5. Teacher rereads and students sketch a diagram or bar model
6. Students solve problem and show work
7. Students write equations and rewrite statement
STEPS for Model Drawing…

Listen and Sketch - Project GLAD

1. Fold paper into four sections

<table>
<thead>
<tr>
<th>Understand the Problem:</th>
<th>Plan/Sketch:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Solve:</th>
<th>Check:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Equation</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Answer Statement:</th>
<th>MMM statement</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Adrian, Ben and Christy put their money together to buy a birthday present for their mother. They had $78.75 altogether. Ben had half as much as Christy and Adrian had 4 times as much as Ben. How much money did Christy contribute?
Jose collected _____ sports cards. He had ____ more baseball cards than football cards. How many baseball cards did Jose have?

Mmm Statement:
Jose collected 425 sports cards. He had 75 more baseball cards than football cards. How many baseball cards did Jose have?

Mmm Statement:
Listen and Sketch - Project GLAD
Baseball Problem

<table>
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<tr>
<th>Understand the Problem:</th>
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<th>Solve:</th>
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<th>Answer Statement</th>
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Addition and Subtraction Problem Types

Understand the different addition and subtraction problem types.

- Dark Gray reflects problem types in Kindergarten
- Light Gray reflects problem types in first grade
- Second grade is responsible for all problem types

- Beyond second grade problem types are reflected in multistep problems, fractions, and decimals.
Addition and Subtraction Problem Types

<table>
<thead>
<tr>
<th>Table 2: Addition and subtraction situations by grade level.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Add To</strong></td>
</tr>
<tr>
<td><strong>Result Unknown</strong></td>
</tr>
<tr>
<td>A bunnies sat on the grass. B more bunnies hopped there. How many bunnies are on the grass now?</td>
</tr>
<tr>
<td>[ A + B = \square ]</td>
</tr>
<tr>
<td><strong>Change Unknown</strong></td>
</tr>
<tr>
<td>A bunnies were sitting on the grass. Some more bunnies hopped there. Then there were C bunnies. How many bunnies hopped over to the first A bunnies?</td>
</tr>
<tr>
<td>[ A + \square = C ]</td>
</tr>
<tr>
<td><strong>Start Unknown</strong></td>
</tr>
<tr>
<td>Some bunnies were sitting on the grass. B more bunnies hopped there. Then there were C bunnies. How many bunnies were on the grass before?</td>
</tr>
<tr>
<td>[ \square + B = C ]</td>
</tr>
<tr>
<td><strong>K-5 Progression on Counting and Cardinality and Operations and Algebraic Thinking</strong></td>
</tr>
<tr>
<td><strong>Take From</strong></td>
</tr>
<tr>
<td><strong>Total Unknown</strong></td>
</tr>
<tr>
<td>C apples were on the table. I ate B apples. How many apples are on the table now?</td>
</tr>
<tr>
<td>[ C - B = \square ]</td>
</tr>
<tr>
<td><strong>Both Addends Unknown</strong></td>
</tr>
<tr>
<td>C apples were on the table. I ate some apples. Then there were A apples. How many apples did I eat?</td>
</tr>
<tr>
<td>[ C - \square = A ]</td>
</tr>
<tr>
<td><strong>Addend Unknown</strong></td>
</tr>
<tr>
<td>Some apples were on the table. I ate B apples. Then there were A apples. How many apples were on the table before?</td>
</tr>
<tr>
<td>[ \square - B = A ]</td>
</tr>
<tr>
<td><strong>Put Together /Take Apart</strong></td>
</tr>
<tr>
<td><strong>Difference Unknown</strong></td>
</tr>
<tr>
<td>A red apples and B green apples are on the table. How many apples are on the table?</td>
</tr>
<tr>
<td>[ A + B = \square ]</td>
</tr>
<tr>
<td><strong>Bigger Unknown</strong></td>
</tr>
<tr>
<td>Grandma has C flowers. How many can she put in her red vase and how many in her blue vase?</td>
</tr>
<tr>
<td>[ C = \square + \square ]</td>
</tr>
<tr>
<td><strong>Smaller Unknown</strong></td>
</tr>
<tr>
<td>C apples are on the table. A are red and the rest are green. How many apples are green?</td>
</tr>
<tr>
<td>[ A + \square = C ]</td>
</tr>
<tr>
<td>[ C - A = \square ]</td>
</tr>
<tr>
<td><strong>Compare</strong></td>
</tr>
<tr>
<td><strong>“How many more?” version.</strong> Lucy has A apples. Julie has C apples. How many more apples does Julie have than Lucy?</td>
</tr>
<tr>
<td>[ A + \square = C ]</td>
</tr>
<tr>
<td>[ C - A = \square ]</td>
</tr>
<tr>
<td><strong>“More” version suggests operation.</strong> Julie has B more apples than Lucy. Lucy has A apples. How many apples does Julie have?</td>
</tr>
<tr>
<td>[ A + B = \square ]</td>
</tr>
<tr>
<td><strong>“Fewer” version suggests wrong operation.</strong> Lucy has B fewer apples than Julie. Lucy has A apples. How many apples does Lucy have?</td>
</tr>
<tr>
<td>[ C - B = \square ]</td>
</tr>
<tr>
<td>[ \square + B = C ]</td>
</tr>
</tbody>
</table>
Multiplication and Division Problem Types

- Understand the different problem types...

- Ensure your students have access to each problem type...
## Multiplication and Division Problem Types

<table>
<thead>
<tr>
<th>Unknown Product</th>
<th>Group Size Unknown (&quot;How many in each group?&quot; Division)</th>
<th>Number of Groups Unknown (&quot;How many groups?&quot; Division)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3 \times 6 = ?$</td>
<td>$3 \times ? = 18$ and $18 \div 3 = ?$</td>
<td>$? \times 6 = 18$ and $18 \div 6 = ?$</td>
</tr>
</tbody>
</table>

### Equal Groups

**Measurement example.** You need 3 lengths of string, each 6 inches long. How much string will you need altogether?

**Measurement example.** You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?

**Measurement example.** If 18 plums are to be packed 6 to a bag, then how many bags are needed?

**Measurement example.** You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?

### Arrays*

**Area example.** What is the area of a 3 cm by 6 cm rectangle?

**Area example.** A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?

**Area example.** A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?

### Compare

**Measurement example.** A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?

**Measurement example.** A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?

**Measurement example.** A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?

### General

$\text{General}$

$a \times b = ?$

$a \times ? = p$ and $p + a = ?$

$? \times b = p$ and $p + b = ?$
Time to Practice…

- Practice Problems from Math Model Drawing (PDF)…
- Model Drawing for Challenging Word Problems…
- Specific Grade Level Problems…
Additional Resources:

- Step By Step Model Drawing
- Model Drawing for Challenging Word Problems
- Seminar on CD ROM
- Word Problems for Model Drawing Practice
  - Grades 1-5
Sample Problem: Try It Again!!

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